Relativistic Electron Beam Instability in an Inhomogeneous Cold Plasma

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ABSTRACT

The interaction of relativistic electron beam (REB) with an inhomogenous cold plasma on the spatial growing of beam-plasma instability has been studied. The variation in the plasma density does have a profound effect on the spatial beam-plasma instability. Besides, relativistic effect leads to more power absorption from the relativistic electron beam so REB causing a resonant increase of the electric field. This work may have its interest due its link to HF heating of plasmas.

Key Words: Relativistic Beam Plasma Instability / Plasma Heating

INTRODUCTION

Relativistic electron beam (REB) plasma interaction is a subject of interest in several fields of fusion research. An example of direct application of REB is plasma heating in open-ended system[1-4]. Examples of the indirect use are the creation of high power microwaves[5], generation of beam stabilized compact toroids[6,7] or beam sustained tokamaks[8].

The problem of no relativistic electron beam linear interaction with cold unmagnatized plasma was studied by many authors[9,10]. In these cases, a beam-plasma interaction takes the form of an amplification of waves by beam. It is shown that due to the resonance rise of the plasma is finite and independent of the value of dissipation. In this case the beam not only amplifies waves in the plasma, but also provides for effective absorption of these waves by the methods via plasma stability, amplification and generation of electromagnetic waves, acceleration of charged particles in plasma, high frequency heating of plasma and so on[11-13].

In the present work we investigate the influence of the variable cold plasma density and REB (under the condition of the smallness of phase velocity of waves compared to beam velocity) on the quenching of the beam-plasma instability. We also consider a semi-infinite beam –plasma system ($x \geq x_o$) , in which the unperturbed plasma density $n_o(x)$ is an arbitrary function of x. We shall suppose that ions are sleeping and that the relativistic electron beam is cold and homogeneous.

FUNDAMENTAL WAVES

For simplicity, we consider the case of one –dimensional electrostatic oscillations when the direction of beam propagation, plasma density gradient and wave electric field coincide with the x-axis. We also consider that phase velocity of waves is much less compared to the electron beam velocity.

The initial linearized set of equations (the continuity equation and equation of motion) describing the oscillations in 1-D, for both plasma and electron beam, has the following form:

$$\frac{\partial n_P}{\partial t}+\nabla.[n_P\vec{V}_P]=0 \tag{1}$$

$$V_b=(V_{0b}+V_{1b})e$$

$$\frac{\partial \vec{V}_P}{\partial t}=-\frac{e}{m}\vec{E}-\nu\vec{V}_P, \tag{2}$$

$$\frac{\partial n_b}{\partial t}+\nabla.[n_b\vec{V}_b]=0, \tag{3}$$

$$\frac{\partial (m\vec{V}_b)}{\partial t}+\vec{V}_b.\nabla[m\vec{V}_b]=-eE, \tag{4}$$

Where the subscripts 0 and 1 denotes the unperturbed and first order perturbation quantities respectively, while subscripts b and P refer the beam and plasma parameters respectively . and ν is the collision of plasma electrons with other plasma particles. All other terms have their usual meaning.

Using the Poisson's equation

$$\frac{dE}{dx}=-4\pi e(n_P+n_b), \tag{5}$$

one can reduce equations (1)-(4) to a single second order differential equation, which can be integrated to yield

$$\frac{d^2F(x)}{dx^2}+\kappa^2F(x)=0 \tag{6}$$

where

$$F(x)=k^{-2}(x)E(x)e^{\frac{-i\omega}{V_{ob}}x} \tag{7}$$

$$\tilde{\omega}=\omega+i\nu,$$

$$\kappa^2(x)=\frac{\omega^2{}_{Rb}}{V^2{}_{0b}}\frac{\omega\tilde{\omega}}{(\omega\tilde{\omega}-\omega^2{}_{P_e})},\omega_{Rb}^2=\gamma^2\omega_b^2,\gamma=(1-\frac{V_{0b}^2}{c^2})^{\frac{1}{2}},\omega^2_{P_e,b}=\frac{4\pi e^2n_{0,(P_e,b)}(x)}{m} \tag{8}$$

The term γ in the L H. S. of equation (6) represent the relativistic effect. An equation similar to (6) has been obtained in the past by many authors[11,12] , however the relativistic effect γ, which is of importance for analysis of plasma instability and heating, is neglected.

Following Bohmer, et al., 1973[11], the solution of equation (6) in the region $x\le 0$ gives the following spatially growing modes (upstream) :

$E(x,t)=E_1(x)\exp i(kx-\omega t),\quad (\operatorname{Im}k_1<0)$

where, $k_1=(\omega/V_{0b})+\kappa_1,\quad \kappa_1$ is given by relation (8) in region $x<0$

The most important mode is the one for which $|\operatorname{Im}\kappa_1(\omega)|$ is a maximum. Providing the discontinuity at $x=0$ has no influence on the solution in region $x\prec 0$, we can derive the following solution of equation (6) in the regions $x\le 0$ and $x>0$:

$$F_1 = A_1 e^{i\kappa_1 x} \quad ; \quad x \le 0$$
$$F_2 = A_2 e^{i\kappa_2 x} + A_3 e^{-i\kappa_2 x} \quad ; \quad x \ge 0$$

where both $\operatorname{Im}\kappa_1$ and $\operatorname{Im}\kappa_2$ are negative.

The constants of integration A_i *(i = 1-3)* are determined under the boundary conditions that both F and $\frac{dF}{dx}$ are continuous at $x = 0$, hence,

$$A_2 = \frac{1}{2}\frac{\kappa_1 + \kappa_2}{\kappa_2} A_1, \qquad A_3 = \frac{1}{2}\frac{\kappa_2 - \kappa_1}{\kappa_2} A_1$$

Using the definition (7) the electric field $E_2(x)$ is derived in terms of $E_1(0)$ as:

$$E_2(x) = \{(\frac{E_1(0)\kappa_2}{2\kappa_1^2})[\kappa_1 + \kappa_2]e^{i\kappa_2 x} + (\kappa_2 - \kappa_1)e^{-i\kappa_2 x}\}e^{i\frac{\omega}{V_{0b}}x} \tag{9}$$

$E_2(x)$ yields a power of the form:

$$\begin{aligned}|E_2(x)|^2 &= [|E_1(0)|^2|\kappa_2|^2/4|\kappa_1|^4][|\kappa_2+\kappa_1|^2 e^{i(\kappa_2-\kappa_2^*)x} + |\kappa_2-\kappa_1|^2 e^{-i(\kappa_2-\kappa_2^*)x}] + \\ &2(|\kappa_2|^2 - |\kappa_1|^2)\cos(\kappa_2+\kappa_2^*)x - 2i(\kappa_2\kappa_1^* - \kappa_1\kappa_2^*)\sin(\kappa_2+\kappa_2^*)x\end{aligned} \tag{10}$$

The 3rd and 4th terms on the R. H. S. of (10) are due to the mixing (spatial beats) between the growing and decaying modes in the region $x \ge 0$. It is clear that power is strongly affected by both mixing and relativistic effect. Mixing produce a noticeable effect on $|E_2(x)|^2$ under the conditions $\kappa_1 \ne \kappa_2$; $|\operatorname{Re}\kappa_2| \succ\succ |\operatorname{Im}\kappa_2|$ which are necessary in order for the trigonometric terms in (10) to vary rapidly compared with the exponential growth terms. The (*) represent the conjugate values. To see what type of discontinuity may produce by this effect, we note that $\kappa(\omega, \omega_{P_e})$ and equation (8) yields

$$(\frac{V_{0b}}{\omega_{Rb}})\kappa = (2A)^{-\frac{1}{2}}\{\operatorname{sgn}(\omega)[B + (B^2 + v^2\omega^2\omega_{P_e}^4)^{\frac{1}{2}}]^{\frac{1}{2}} - i[-B + (B^2 + v^2\omega^2\omega_{P_e}^4)^{\frac{1}{2}}]^{\frac{1}{2}}\}, \tag{11}$$

where,

$$A = (\omega^2 - \omega_{P_e}^2)^2 + \omega^2 v^2, \quad B = \omega^2(\omega^2 - \omega_{P_e}^2 + v^2)$$

which completely independent from thermal effect. Here, it is important to consider κ as a function of $\omega_{P_e}^2$, with w held constant.

From (10), we get:

$$|E_2(0)|^2 = |\kappa_2/\kappa_1|^4 |E_1(0)|^2 \tag{12}$$

So that electric field is discontinuous at $x = 0$.

Let's now analyze the solution (9) for a realistic plasma model, i.e., inhomogeneous plasma with a finite gradient in $n_o(x)$. For this we assume:

$$\omega_{P_e}^2(x) = \omega_{P_1}^2(x)[1 + \varepsilon(x/L)]; \qquad (L \ge x \ge 0; \varepsilon \succ -1) \tag{13}$$

Corresponding to a constant density gradient in the transition region. It can be shown that the linear approximation is valid in this case provided that

$$1 \succ\succ |\varepsilon|(\omega_{Rb}/\omega_{P_e})^2(\omega_{P_e}/v)(V_{0b}/\omega_{P_e}L) \tag{14}$$

which indeed requires that $L \ne 0$.

Taking into consideration collision plasma, and according to the experiment of Bohmer, et al. (1973)[11], $(\omega_b/\omega_{P_e})^2 \approx 10^{-4}$, $\omega_{P_e}/v \approx 20$ and $\lambda = V_{0b}/\omega_{P_e} \cong 1cm$, if

$L \succ\succ 10^{-2}\lambda$ (λ is the wavelength of the wave in the homogeneous region), the linear approximation is valid, but does not prove the correctness of solution (9). In order to prove that expression (9) is essentially correct, a solution of wave equation (6) requires using the density profile (13), which yields the equation

$$\frac{d^2F}{d\xi^2} + b^{-2}\xi^{-1}F = 0 \tag{15}$$

where;

$$\xi = a - b(\frac{\omega_{Rb}}{V_{0b}}x); \qquad a = \frac{\omega\tilde{\omega} - \omega_{P_1}^2}{\omega\tilde{\omega}}; \qquad b = \frac{\varepsilon}{L}\frac{V_{0b}\omega_{P_1}^2}{\omega\tilde{\omega}\omega_{Rb}}$$

Solution of (15) is:

$$F(z) = AzJ_1(z) + BzN_1(z); \quad 0 \le x \le L \tag{16}$$

where $z = \frac{2\xi^{1/2}}{b}$ and $J_1(z)$; $N_1(z)$ are the Bessel function of the first and second kind respectively.

Solution of (16) is an agreement with that obtained by Bohmer, et al., except the presence of a new term γ on L. H. S. which strongly proportional to the REB.

$$F(x) = \sum_{\pm} A_{\pm}e^{\pm ik_2(x-L)}; \quad x \ge L$$

where

$$\begin{aligned} A_{\pm} = \frac{\pi}{4}z_1F(0)\{&[(N_0(z_0)J_1(z_1) - J_0(z_0)N_1(z_1)) \pm (J_1(z_0)N_0(z_1) - N_1(z_0)J_0(z_1))] + \\ &i[(N_1(z_0)J_1(z_1) - J_1(z_0)N_1(z_1)) \pm (J_0(z_1)N_0(z_0) - N_0(z_1)J_0(z_0))]\} \end{aligned} \tag{17}$$

Such that

$z_0 = \frac{2\omega_{Rb}}{bV_{0b}}\frac{1}{\kappa_1}$; $z_1 = \frac{2\omega_{Rb}}{bV_{0b}}\frac{1}{\kappa_2}$ are correspond to ($x = 0, x = L$) and:

$$\kappa_1 = \frac{\omega_{Rb}\omega}{V_{0b}\sqrt{\omega\tilde{\omega} - \omega_{P_1}^2}}; \qquad \kappa_2 = \frac{\omega_{Rb}\omega}{V_{0b}\sqrt{\omega\tilde{\omega} + \omega_{P_1}^2(1+\varepsilon)}}$$

Equation (17) can be re-written as

$$A_{\pm} = (\frac{\omega_{Rb}}{bV_{0b}})^2\frac{F(0)}{k_1k_2^2}[(\kappa_1 \mp \kappa_2) \pm i\frac{V_{0b}}{\omega_{Rb}}b\kappa_1\kappa_2]\ln\frac{\kappa_2}{\kappa_1}$$

The case of interest is when ε is not too small and $(\frac{L}{\lambda})$ is not very large [large rapid changes in $n_0(x)$] which is the opposite extreme from the WKB situation. From the definitions following (15) we note that $\max(\nu/\omega_{P_e};\varepsilon) \succ |\xi|$ and $|b| \approx \varepsilon(\omega_{P_e}/\omega_{Rb})(\lambda/L)$, where $\lambda = V_{0b}/\omega_{P_e}$. Therefore, if $(\frac{L}{\lambda})$ is not too large, and ε is not too small, then b is large and ξ will be fairly small. Consequently, z is small in this case and Bessel function in (17) may be expanded for small argument. When this is done, one finally obtain the approximate result

$$E_2(x) = (\frac{\omega_{Rb}}{bV_{0b}})^2\frac{F(0)}{k_1k_2^2}[(\kappa_1 \mp \kappa_2) \pm i\frac{V_{0b}}{\omega_{Rb}}b\kappa_1\kappa_2]\ln\frac{\kappa_2}{\kappa_1}e^{\pm i\kappa_2(x-L)+\frac{i\omega}{V_{0b}}x} \tag{18}$$

where $x \ge L$.

The term ω_{Rb} in the R. H. S. of equation (18) represent the relativistic effect due to REB (i. e., increase in electric field intensity).

The result (18) may be compared with the result (9) for the simple discontinuous model. It can be see that provided b is large and (14) is satisfied (L not too large and not too small), the result (9) is a good approximation to equation (18).

CONCLUSIONS

Relativistic electron beam (REB) leads to wave amplification and accordingly to plasma heating in beam-plasma system (solutions (9) and (18)). From (18), we could conclude that power absorbed from the beam into plasma is strongly affected by both mixing and REB. The variation in the plasma density does have a profound effect on spatial beam-plasma instability. This effect indicates that the resulting drop in intensity of electric field is a sensitive function of the plasma discontinuity. It also growing modes that only if the plasma density decreases.

REFERENCES

(1) L. E. Thode, Phys. Fluids, 19, 831 (1976).

(2) D. A. Hammer, K. A. Gerber, and A. W. Ali, IEEE Trans. Plasma Sci. PS-7, 83 (1979).

(3) P. H. de Haan, G. C. A. M. Janssen, H. J. Hopman, and E. H. A. Granneman, Phys. Fluids, 25, 592 (1982).

(4) R. L. Sheffield, H. A. Davis, C. A. Ekdahl, and M. D. Montgomery, Proceedings of the 1982 International Conference on Plasma Physics, Goteborg, Sweden, 9-15 June, 85 (1982).

(5) S. N. Voronkov, V. I. Krementsov, P. S. Strelkov, and A. G. Shkvarunets, in Proceedings of the Tenth European Conference on controlled Fusion and Plasma Physics (USSR Academy of Sciences, Moscow, 1981), 11, M6 (1981).

(6) J. D. Sethian, K. A. Gerber, A. W. de Silva, and A. E. Robson, Bull. Am. Phys. Soc., 27, 5E1(1982).

(7) R. Jayakumar, M. Parker, D. Taggart, H. J. Hopman, and H. H. Fleischmann, Bull. Am. Phys. Soc., 27, 1W5 (1982).

(8) A. Mohri, K. Narihara, Y. Tomita, T. Tsuzuki, M. Hasegawa, and K. Ikuta, Nucl. Fusion Suppl. 1, 511 (1981).

(9) W. H. Amein, V. V. Dolgopolov, A. M. Hussein, and K. E. Zayed, Plasma Physics, 17, 479 (1975).

(10) A. A. Ivanov, N. G. Popkov, J. Wilhelm, R. Winkler, Beitr. Plasma Phys., 7, 23 (1984).

(11) H. Bohmer, E. A. Jackson and Rather, Phys. Fluids, 16, 1064 (1973).

(12) M. V. Kuzelev and A. A. Rukhadze, "Elecrtodynamics of Dense Electron Beam in Plasma" Mosco, "Nauka" (1990).

(13) M. V. Kuzelev and A. A. Rukhadze, English Completed Edition "Plasma Free Electron Lasers" "Edition Frontier" Paris (1995).